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ence is not a matter of belief, but a scientific certainty. All former proofs of the existence of God were necessarily failures, because in all cases the attempt was made to prove the existence of an anthropomorphic God with arguments that prove the true God, the eternal norm of being, and here the argument breaks down, because it no longer applies to the idea of an anthropomorphic God.

Leibniz has not overcome the mystical conception both of God and truth. He has unfortunately adopted the very primitive conception of an atomic nature of reality which is described in his monadology. It is strange that a man of his caliber did not see how contradictory is the idea of God as the central monad. On the other hand his theory is vindicated if we interpret his God to be the universal and omnipresent norm that regulates every event and constitutes the cosmic order of the world.

Insisting on the unity of the soul, Leibniz conceived all unities as local units, and these innumerable local units, the monads, were conceived as centers of force endowed with feeling and an entelechy, which means that they were capable of pursuing purposes. At the same time Leibniz held them to be separate entities, so as to render their cohesion and interaction a profound problem which could be solved only by the bold hypothesis of the preestablished harmony.

The problem of unity together with all problems of combination and configuration belongs in the domain of pure form. Combination of several parts working in cooperation constitute a unity and introduce something new. It did not exist before and will break to pieces again, but the law of its combination remains forever and constitutes the eternal background of its existence. The sensationalist school misses the main point of all philosophical considerations and thus loses the essence of the significance of religion; but Leibniz who discovers the weak spot in their arguments has not succeeded in presenting a satisfactory solution of the problem but ends in proclaiming a mystical God-conception and a dogmatic proclamation of a preestablished harmony.

P. C.

EXISTENTS AND ENTITIES.¹

That we must distinguish between what we may call "having existence" and "having entity or being" becomes evident when we look somewhat closely at ordinary mathematical propositions. A class (or system, or aggregate) *M* is said to "exist" when it has

¹ Cf. *Monist*, Jan. 1910, Vol. XX, p. 114, note 85.

at least one member;² whereas, when mathematicians speak of, for example, "the existence of roots of an equation" or "the existence of the definite integral of a continuous function," they use the word "existence" in another sense: the roots or the integral are not classes, but individuals constructed out of mathematical concepts to supply an answer to certain questions. We can, of course, consider such an individual as *the* member of the class (N) whose sole member is this individual, and can then consider the second kind of mathematicians' existence-proofs as proofs of the existence of the class N; but we should, for the sake of clearness, avoid speaking of the "existence" of the member³ of N, and use some such word as "entity" or "being" instead.

Mr. B. Russell⁴ has thus distinguished *being* and *existence* in 1901: "*Being* is that which belongs to every conceivable term, to every possible object of thought—in short to everything that can possibly occur in any proposition, true or false, and to all such propositions themselves. Being belongs to whatever can be counted. If A be any term that can be counted as one, it is plain that A is something, and therefore that A is. 'A is not' must always be either false or meaningless. For if A were nothing, it could not be said not to be; 'A is not' implies that there is a term A whose being is denied, and hence that A is. Thus unless 'A is not' be an empty sound, it must be false—whatever A may be, it certainly is. Numbers, the Homeric gods, relations, chimeras, and four-dimensional spaces all have being, for if they were not entities of a kind, we could make no propositions about them. Thus being is a general attribute of everything, and to mention anything is to show that it is.

"*Existence*, on the contrary, is the prerogative of some only amongst beings. To exist is to have a specific relation to existence—a relation, by the way, which existence itself does not have. This shows, incidentally, the weakness of the existential theory of judgment—the theory, that is, that every proposition is concerned with something that exists. For if this theory were true, it would still be true that existence itself is an entity, and it must be admitted that existence does not exist. Thus the consideration of existence itself

² Cf., e. g., Dedekind, *Was sind und was sollen die Zahlen?* 2d ed., Braunschweig, 1893, pp. 5, 12; or *Essays on the Theory of Numbers*, Chicago, 1901, pp. 49, 58; Russell, *The Principles of Mathematics*, Cambridge, 1903, pp. 21, 32.

³ Of course, the member of N may be itself a class and may thus "exist," but we obviously need not consider this further.

⁴ *Mind*, N. S., Vol. X, No. 39, 1901, pp. 310-311.

leads to non-existential propositions, and so contradicts the theory...."

This doctrine was repeated in Mr. Russell's *Principles of Mathematics*;⁵ the existence-theorems of mathematics were said⁶ to be "proofs that the various classes defined are not null," and the earlier statement⁷ that these theorems are proofs "that there are entities of the kind in question" must not be taken to mean what it apparently expresses.

While Mr. Russell emphasized the distinction between *entity* and *existence*, it does not seem that at that time he quite realized the full bearings of the question, at least in mathematics. He attributed a denotation to every term that can possibly occur in a proposition. Thus "the round square" had a denotation, and the only further existence-question in logic and mathematics was whether the numbers—at least such as were defined as classes—, classes of spaces, and so on, could be proved to "exist,"—whether members of the classes in question could be constructed by logical methods provided that the initial postulates are granted.

* * *

Before going on to discuss the clear separation of the important question of entity from the less important question of existence, which came in Mr. Russell's later works, we will refer to the very strong tendency, even among logicians and mathematicians, to attribute a denotation to every denoting phrase.

Thus, H. MacColl⁸ remarked that a symbol which corresponds to nothing in our universe of admitted realities, has, nevertheless, "like everything else named," a *symbolical* entity. In his sixth paper on "Symbolic Reasoning,"⁹ MacColl attempted to give a simple theory of the existential import of propositions.

By $e_1, e_2, e_3, \dots$, he denoted "our universe of *real existences*," and by $o_1, o_2, o_3, \dots$, "our universe of *non-existences*, that is to say, of *unrealities*, such as *centaurs, nectar, ambrosia, fairies*, with self-contradictions, such as *round squares, square circles, flat spheres*,

⁵ Pp. 449-450; cf. pp. 43, 71.

⁶ *Ibid.*, p. 497.

⁷ *Ibid.*, p. vii.

⁸ *Symbolic Logic and its Applications*, London, 1906, p. 42; MacColl here and elsewhere used the word "existence" where we use "entity." Cf. *Mind*, N. S., Vol. XI, 1902, pp. 356-357.

⁹ *Mind*, N. S., Vol. XIV, 1905, pp. 74-81; cf. *Symbolic Logic and its Applications*, pp. 5, 76-78.

etc.”; the “symbolic universe, or universe of discourse,” *S*, may consist either wholly of realities, wholly of unrealities, or partly of realities and partly of unrealities. . . . If *A* denotes an individual or a class, any intelligible statement $\phi(A)$ containing the symbol *A*, implies that the individual or class represented by *A* has a *symbolic* existence; but whether the statement $\phi(A)$ implies that that which *A* denotes has a *real* or *unreal* or (if a class) partly real and partly unreal existence, depends upon the context.”

We will pass over the discussion between Messrs. MacColl and A. T. Shearman¹⁰ on the interpretation of the Boolean equation “ $O=OA$,” and come to Mr. Russell’s articles of 1905,¹¹ in which the theory of non-entity was, it seems, for the first time treated satisfactorily.

The sense in which the word “existence” is used in symbolic logic is a definable and purely technical sense. To say that *A* exists means that *A* is a class which has at least one member. Thus whatever is not a class does not exist in this sense; and among classes there is just one that does not exist, namely, the null-class. MacColl’s two universes of existences and non-existences are not to be distinguished in symbolic logic, and each of them is identical with the null-class. There are no centaurs; “*x* is a centaur” is false whatever value we give to *x*, even when we include values which do not “exist” in the meaning which occurs in philosophy and daily life, such as numbers or propositions.

“The case of nectar and ambrosia is more difficult, since these seem to be individuals, not classes. But here we must presuppose definitions of nectar and ambrosia: they are substances having such and such properties, which, as a matter of fact, no substances do have. We have thus merely a defining concept for each, without any entity to which the concept applies. In this case, the concept is an entity, but it does not denote anything. . . . These words [such as nectar and ambrosia] have a *meaning*, which can be found by looking them up in a classical dictionary, but they have not a *denotation*: there is no entity, real or imaginary, which they point out.”

¹⁰ *Mind*, N. S., Vol. XIV, 1905, pp. 78-79, 295-296, 440, 578-580; Vol. XV, 1906, pp. 143-144; and Shearman’s book *The Development of Symbolic Logic; a Critical-Historical Study of the Logical Calculus*, London, 1906, pp. 161-171.

¹¹ “The Existential Import of Propositions,” *Mind*, N. S., Vol. XIV, 1905, pp. 398-401; “On Denoting,” *ibid.*, pp. 479-493.

The last sentence refers to Frege's¹² distinction of *Sinn* (meaning) and *Bedeutung* (denotation).

A point of passing interest in connection with an attempt at the solution of a mathematical paradox, referred to later, is this sentence in MacColl's reply:¹³ "I may mention, as a fact not wholly irrelevant, that it was in the actual application of my symbolic system to concrete problems that I found it absolutely necessary to label realities and unrealities by special symbols *e* and *o*, and to break up the latter class into separate individuals, *o*₁, *o*₂, *o*₃, etc., just as I break up the former into separate individuals *e*₁, *e*₂, *e*₃, etc."

When a phrase which in form is denoting, and yet does not denote anything,—e. g., "the present king of France,"—occurs in the statement of a proposition, the question as to the interpretation of propositions in whose verbal expression this phrase occurs arises, and Mr. Russell, in the article "On Denoting" referred to, succeeded in assigning a meaning to every proposition in whose verbal expression any denoting phrases—whether they appear to denote something or nothing at all, e. g., everything, nothing, something, a man, every man, no man, the father of Charles II, the present king of France—occur. It is not necessary to assume that denoting phrases ever have any meaning in themselves.

The theory of MacColl and the allied theory of Meinong were rejected by Mr. Russell¹⁴ because they conflict with the law of contradiction. If any grammatically correct denoting phrase stands for an *object* although such objects may not *subsist*, such objects are apt to infringe the law of contradiction. Thus it is contended that the round square is round, and also not round.

To solve the paradoxes that appear in the mathematical theory of aggregates, Mr. Russell treated classes and relations in the same way as he treated denoting phrases.¹⁵

Poincaré, among others, recognized that all the paradoxes of the modern theory of aggregates, such as those of Burali-Forti, Russell and Richard, arise from a kind of vicious circle which may be expressed, in the language of Peano, thus: Everything which

¹² "Ueber Sinn und Bedeutung," *Zeitschr. für Phil. und phil. Kritik*, Vol. C, 1892, pp. 25-50.

¹³ *Mind*, N. S., Vol. XIV, 1905, p. 401.

¹⁴ *Ibid.*, pp. 491, 482-483.

¹⁵ "On Some Difficulties in the Theory of Transfinite Numbers and Order Types," *Proc. Lond. Math. Soc.* (2), Vol. IV, 1906, pp. 29-53 (cf. especially the part on the "No-Classes Theory"); "Les Paradoxes de la Logique," *Rev. de Métaphys. et de Morale*, Vol. XIV, 1906, pp. 627-650.

contains an apparent variable must not be one of the possible values of this variable.¹⁶ But Poincaré did not perceive that if we wish to avoid such vicious circles we must have recourse to a fundamental re-moulding of logical principles, more or less analogous to the "no classes" theory. To have shown this seems to be one of Mr. Russell's greatest merits; simply because practically all the other mathematicians who have interested themselves in the paradoxes did not realize this important fact. Thus, said Mr. Russell,¹⁷ the method by which Poincaré tried to avoid the vicious circle consists in saying that when we assert that "all propositions are true or false," which is the law of the excluded middle, we exclude tacitly the law of the excluded middle itself. The difficulty is to make this tacit exclusion legitimate without falling into the vicious circle. If we say, "All propositions are true or false, excepting the proposition that every proposition is true or false," we do not avoid the vicious circle. For this is a judgment bearing on all propositions, viz.: "All propositions are either true or false, or identical with the proposition that all propositions are true or false." And that supposes that we know the meaning of "all propositions are true or false," where *all* has no exception. That comes to defining the law of the excluded middle by: "All propositions with the exception of the law of the excluded middle are true or false," where the vicious circle is flagrant. We must, then, find a means to formulate the law of the excluded middle in such a way that it does not apply to itself.

On the details of the new construction of logic in such a way that the paradoxes are avoided while nearly all of the work of Cantor on the transfinite is preserved, we must refer to Mr. Russell's works of 1908 and 1910.¹⁸ Mr. Russell's method of avoiding the paradoxes in question is by what he called the "theory of types," and the object of this theory was shortly described by Dr. Whitehead and Mr. Russell¹⁹ as follows: "The vicious circles in question arise from supposing that a collection of objects may contain members which can only be defined by means of the collection as a

¹⁶ We may also express this principle as follows: A collection of objects may not contain members which can only be defined by means of the collection as a whole.

¹⁷ *Rev. de Métaphys. et de Morale*, Vol. XIV, pp. 644-645.

¹⁸ "Mathematical Logic as Based on the Theory of Types," *Amer. Journ. of Math.*, Vol. XXX, 1908, pp. 222-262; A. N. Whitehead and B. Russell, *Principia Mathematica*, Vol. I, Cambridge, 1910, pp. 39-88.

¹⁹ *Op. cit.*, p. 39.

whole. Thus, for example, the collection of *propositions* will be supposed to contain a proposition stating that 'all propositions are either true or false.' It would seem, however, that such a statement could not be legitimate unless 'all propositions' referred to some already definite collection, which it cannot do if new propositions are created by statements about 'all propositions.' We shall, therefore, have to say that statements about 'all propositions' are meaningless. More generally, given any set of objects such that, if we suppose the set to have a total, then such a set cannot have a total. By saying that a set has 'no total,' we mean, primarily, that no significant statement can be made about 'all its members.' Propositions, as the above illustration shows, must be a set having no total. The same is true, as we shall shortly see, of propositional functions, even when these are restricted to such as can significantly have as argument a given object *a*. In such cases, it is necessary to break up our set into smaller sets, each of which is capable of a total. This is what the theory of types aims at effecting."²⁰

* * *

In the next place, we shall go back four or five years in time, and see how the distinction between *entity* and *existence* became necessary in a mathematical investigation which is somewhat familiar to me. If I consider, at rather greater length than it deserves, my own work of 1903 and 1904²¹ on the contradiction of Burali-Forti and its bearings on the theory of well-ordered aggregates, it is merely because familiarity with this investigation enables me to point out a small, unobserved merit which it has, in distinguishing *entity* from *existence*, and also to give yet another illustration of the tendency—which seems particularly common with mathematicians—of holding to the belief in the being or existence or subsistence in some sense, of a non-entity.

Burali-Forti had found, in 1897, the now well-known contradiction arising from the fact that 'the ordinal type of the whole series of (finite and transfinite) ordinal numbers' appears both to be and not to be the greatest ordinal number. From this I concluded, in 1903, that there are no such things as "the type" and

²⁰ The theory of logical types was described, in ordinary language, in *op. cit.*, pp. 39-68; and the theory of denoting was explained in the chapter on "Incomplete Symbols" (*ibid.*, pp. 69-88).

²¹ A general account of these investigations is contained in my paper, written in Peano's international (uninflected) Latin: "De Infinito in Mathematica," in *Revista de Mathematica*, Vol. VIII.

"the cardinal number" of the series just referred to. Hence, by a tacit use of an axiom afterwards stated explicitly by Zermelo, I concluded that every aggregate which has a cardinal number and every series which has a type can be well-ordered. The use of Zermelo's axiom was, with me as with most mathematicians, unrecognized; it occurred in some work of Mr. G. H. Hardy's on which I based my argument; and I was really concerned, not so much with the proof that every aggregate can be well-ordered, as with the proof that the series (W) of ordinal numbers has no type.

The matter becomes simpler to express when we consider *classes* instead of *series*. My contention, then, was that there is no such thing as "the cardinal number of the class of ordinal numbers" seems to represent. But if we adopt, as I adopted, the Frege-Russell definition of the cardinal number of a class u as the class of those classes which are similar to (can be put in a one-one correspondence with) u , there arises a difficulty. The cardinal number of the class w of ordinal numbers is the class of those classes which are similar to w ; and this class certainly *exists*, for we can point out at least one member of it, namely, w itself, for w is similar to w . On the other hand, we have reason to deny that there *is* such a class as the cardinal number of w , and most mathematicians express this by saying that the cardinal number in question does not "exist." Of course, the solution of this apparent contradiction is that "the cardinal number of w " is a phrase denoting nothing—there is no such entity as the cardinal number of w . If it *did* denote a class, that class would be existent.

So, in my above-quoted paper, I distinguished between the existence of a class u from the entity of a thing v . The symbol ' $\exists u$ ' was used, following Peano, to denote that u exists, and the symbol " Ev " was used to denote the proposition that v is an entity. The symbol " Ev " was defined by the definition of "not- Ev " as " v is a member of the null-class." Since the null-class has no members, and is defined as the x 's satisfying a propositional function, such as x is not identical with x , which is always false, this is a most paradoxical way of stating the case about non-entity,²² and the paradox results from the assumption that, in some sense, there *is* a v ,—that, as MacColl would have said, v has a "symbolical

²² On printing the above article, Professor Peano wrote to me, on Jan. 1, 1906, as follows: "... I see the new symbol E, which you do not define symbolically, but the importance of which I believe I have understood.... It would be necessary to introduce many kinds of null-class (Δ): Δ_0 = that of the *Formulaire*; Δ_1 = the class of classes, which has no classes; Δ_2 for the classes

existence." But, as Dr. Whitehead and Mr. Russell²³ pertinently remark: "We cannot first assume that there is a certain object, and then proceed to deny that there is such an object." Russell's solution of the difficulty about propositions asserting that "the so-and-so is not an entity" is to reduce all such propositions to a form not involving the assumption that "the so-and-so" is a grammatical subject. "The so-and-so," whether it appears to denote something or not, is an *incomplete* symbol, like the d/dx of mathematics.

* * *

It has, I trust, been not quite without interest to see how the important distinction of *existence* and *entity* in mathematics struggled into clearness. We have seen before²⁴ that the discussions on "existence" of MM. Poincaré and Couturat were conducted in obscurity. This obscurity was produced by the confusion of the two notions of *existence* and *entity*, and the consequent use of one word to denote both.

When, in a paper published in 1904, I used the badly chosen term "inconsistent" for an aggregate whose cardinal number is a non-entity—"does not exist," I said then—Mr. Russell rightly objected that, given a class u , its cardinal number *must* exist, since u is a member of the class called the cardinal number of u . And yet there was an undoubted difficulty about what I called "inconsistent" classes. We know now that—at any rate when the number of a class is defined logically—it is a delusion that there are such "inconsistent" classes,—they are non-entities. If they *were* entities, their cardinal numbers would "exist."

There is one more thing to be noticed: it is the *entity* of a number that is most important, the proof of its *existence* is less so. In his *Principles* of 1903, Mr. Russell laid great stress on the existence-proofs of numbers and classes of spaces. Let us consider the case of real numbers. A real number is, according to Mr.

of classes; Λ_1 for the classes of classes of classes;.... Λ_n ,.... Λ_w ,.... There is the generation of the transfinite numbers, in the principles of logic. There results this rather laughable consequence, that the new philosophers have decomposed *nothing* into a transfinite number of classes!"

²³ *Op. cit.*, p. 69. We may remark here, as I have done in a review of Whitehead and Russell's *Principia* in the *Cambridge Review* for 1911, that the authors (cf. pp. 32, 69, 182, 229) use the word "existence" ambiguously; though, of course, there is no ambiguity when the proper technical symbols ($\exists$ and E ; E only occurring in a phrase involving incomplete symbols) are used.

²⁴ *Monist*, Jan. 1910, Vol. XX, pp. 113-116.

Russell, a certain class of rational numbers; its existence can be proved, and one feels satisfied. But a rational number or a negative number, being a relation, does not "exist," and yet one would have thought existence quite as important in these cases as in the case of real numbers.²⁵ I hope to go more fully into this question on another occasion.

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IDEALISM AS A FORCE.

A MECHANICAL ANALOGY.

In the present state of knowledge the man of intelligence has much difficulty in deciding what course of conduct he should adopt in regard to beliefs and social and religious practice without at the same time violating these principles which he has obtained from science and critical philosophy. Before venturing to suggest exactly what position he should (and eventually must) take up, a little consideration of the importance of the older ideas and their relation to new ones would be advisable. I propose to introduce various mechanical analogies in this sketch, for two reasons. First, because I think they show forth more clearly the nature of the phenomena described, and second, a training in scientific thought soon shows one that mechanical laws pervade the whole universe, mental, moral and physical. I do not use the word "mechanical" in at all a derogatory sense. As a matter of fact, although it seems at first contrary to our ideas of perfection realized by a continuous process of adjustment, the really perfect state is the mechanical one, where each part has a definite and unchanging relation to all the other parts, so that a change in its condition is accompanied by a change in all other parts in accordance with the nature of that mutual relation. Surely this is what is meant by "correspondence with environment," if there is the proviso of stability. All moral philosophers have more or less directly stated that the key to morality is the Golden Rule, "Do as you would be done by," or as K'ung-fu-tze puts it, in one word, "Reciprocity," i. e., mutual bearing upon one another. This condition of mutual bearing is essentially, when complete, a mechanical

²⁵ Frege (*Grundlagen der Arithmetik*, Breslau, 1884, pp. 114-115) indicated such definitions of all the numbers of analysis as would enable him to prove the existence in every case.